The K^0 condensed phase of color-flavor locked quark matter : its formation as a stable state reexamined

Xiao-Bing Zhang and Xue-Qian Li

Department of Physics, Nankai University, Tianjin 300071, China

Abstract

Taking the vector kaon-quark interaction into account, we reexamine in detail the neutral kaon condensation in color-flavor locked quark matter. It is found that the pairing phenomena in the quark matter are somewhat influenced by K^0 condensation. Different from the previous predictions, we find that the K^0 condensed phase might no longer be more stable than the normal phase even if K^0 condensation can occur in the color-flavor locked matter.

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I. Introduction

Investigations of strongly interacting matter at high baryon density have attracted much attention for years. In recent years, the studies of the physics with dense quark matter become a "hot" topic since some color superconducting phases were proposed for high-density QCD. For the case of three massless flavors, it has been suggested that the original color and flavor $SU(3)_{color} \times SU(3)_L \times SU(3)_R$ symmetries of QCD are broken down to the diagonal subgroup $SU(3)_{color+L+R}$ via BCS like pairing [1]. This symmetry breaking pattern exhibits a resemblance to the ordinary chiral symmetry breaking in the low-density QCD case. Since color and flavor rotations are locked together, such a particular symmetric state is called the color-flavor locked (CFL) phase. It is widely accepted to be a possible ground state of the dense quark matter with three flavors. The main low-energy degree of freedom in this phase is an octet of pseudo Goldstone modes that are associated with the spontaneous chiral symmetry breaking. The effective Lagrangian describing the Goldstone modes in dense quark matter was studied in Refs.[2, 3, 4]. Recently, condensation of the Goldstone modes has been investigated by using the chiral effective Lagrangian [5, 6, 7, 8]. Bedaque and Schäfer indicated that the stress induced by the strange quark mass m_s triggers the condensation of neutral kaon modes [6]. The K^0-mode condensed phase of CFL quark matter, which is called as the CFLK^0 phase in the following, was believed to be more realistic than the normal

CFL phase without condensations [6, 7]. Therefore, the CFLK^0 phase is expected to exist in core of neutron stars and investigations on this phase are relevant to the physics of compact stars [7, 8]. The formation of K^0-mode condensation makes it possible that CFL quark matter reduces its strange content. The hadronic matter at finite density possesses non-zero strangeness because of the presence of hyperons and/or the kaon-meson condensation. From this point of view, investigation on the kaon-mode condensed phases of CFL matter might be necessary for exploring the QCD phase diagram such as the phase transition from nuclear matter to CFL quark matter [9, 10, 11].

In the references cited above, the properties of CFL matter is assumed to be unchanged regardless of whether K^0 condensation occurs or not. Thus the CFLK^0 phase can be understood as a simple " mixture " of normal CFL matter and the system consisting of the condensed kaons, namely CFLK^0=CFL+ K^0. Correspondingly, the phase transition between the CFL and CFLK^0 phases is of second order [6, 7, 8]. Based on this assumption, it seems to be an undoubted conclusion that the CFLK^0 phase is *always* more stable than the CFL phase. Nevertheless, some details concerning the stability of the CFLK^0 phase need to be further investigated. Generally the CFLK^0 phase should be different from the normal CFL phase in all ways. Therefore there might exist some effects of K^0 condensation on the CFL matter in which the condensed kaons emerge. Such effects are expected to manifest once the K^0 condensation with finite condensate strength has appeared.

On the other hand, it is found more recently that the gapless phases of CFL matter might replace the ordinary CFL phases where there is a common pairing gap denoted as Δ [12]. If the electron chemical potential is ignored, the pairing between the green-strange and blue-down quarks might become unstable gradually with the increasing strange quark mass m_s [12]. Thus, the corresponding gapless phase replaces the CFL/CFLK^0 phase as long as m_s is large enough. Noticing that K^0 condensation is caused by the nonzero value of m_s essentially, the question arises naturally whether and how the CFLK^0 phase exists as a stable state if m_s is relatively large. If the assumption of CFLK^0=CFL+ K^0 were correct so that the larger m_s is, the more stable the CFLK^0 phase would be [6, 7]. Then, the CFLK^0 phase would be energetically favorable eventually and even be favored over the gapless phase. Obviously, the above conjecture is contradictory to the existence of the gapless phases. In this sense, the previous treatment for the CFLK^0 phase on the basis of CFLK^0=CFL+ K^0 might be problematic. This discrepancy motivates us to reexamine the mechanism of K^0 condensation and the stability condition for the CFLK^0 phase. Taking an effective kaon-quark interaction into account, we find that the CFL pairing phenomena in the quark matter are influenced by K^0 condensation. As a consequence, the CFLK^0 phase might be no longer energetically

favorable with respect to the CFL phase even if K^0 condensation can occur. Moreover, we suggest that the transition from the CFL phase to the CFLK^0 phase is generally a first order transition, instead of the second order one.

II. K^0 condensation in CFL matter

Let us begin with the description of CFL quark matter when no condensation occurs. The pressure of the CFL phase can be written as

$$\mathcal{P}_{CFL} = \mathcal{P}_{unpair} + \mathcal{P}_{pair}, \tag{1}$$

where $\mathcal{P}_{unpair}$ denotes the pressure from the unpaired quark matter where all the three flavors have the same Fermi momenta and its form is model dependent. The pressure induced by the CFL pairing is given by [10, 13]

$$\mathcal{P}_{pair} = \frac{3\Delta^2\mu^2}{\pi^2}, \tag{2}$$

where μ is the quark chemical potential and Δ is the CFL pairing gap in the color triplet. The baryon density of the CFL phase reads [10]

$$\rho_{CFL} = \frac{p_F^3 + 2\Delta^2\mu}{\pi^2}, \tag{3}$$

and the common Fermi momentum p_F is required to be [13]

$$p_F = 2\mu - \sqrt{\mu^2 + m_s^2/3} \approx \mu - m_s^2/(6\mu), \tag{4}$$

in order to guarantee the electrical neutrality of the CFL quark matter.

For the Goldstone modes in CFL quark matter, the leading terms of the concerned effective Lagrangian are given as [2, 6]

$$\mathcal{L}_G = \frac{f_\pi^2}{4} Tr(\partial_0 U \partial_0 U^+ - v_\pi^2 \partial_i U \partial_i U^+) - A[det(M)Tr(M^{-1}U) + h.c.], \tag{5}$$

where v_π is the 3-velocity of the Goldstone modes, M is the quark mass matrix and the chiral field U is defined by $U = \exp(i\sqrt{2}\Phi/f_\pi)$ with the octet of Goldstone modes Φ. By perturbative calculations at high density QCD, the pion decay constant f_π and the low energy coefficient A are [4]

$$f_\pi^2 = \frac{21 - 8\ln 2}{18}\frac{\mu^2}{2\pi^2}, \tag{6}$$

and

$$A = \frac{3\Delta^2}{2\pi^2}, \tag{7}$$

respectively. By Eqs.(5) and (7), the masses of Goldstone modes are given by

$$m_\pi^2 = \frac{6\Delta^2}{\pi^2 f_\pi^2} m_q m_s, \quad m_K^2 = \frac{3\Delta^2}{\pi^2 f_\pi^2} m_q (m_q + m_s), \tag{8}$$

where $m_q = m_{u,d}$ is the light quark mass in the limit of isospin symmetry. Eq.(8) shows that the kaon-mode is lighter than the pion-mode[1] , which implies that the formation of kaon condensation is easier than the pion condensation in the CFL environment.

In the vicinity of the Fermi surface, quasi-quark excitations are active and thus become the natural degrees of freedom in the CFL effective theory. The interaction among these quasi quarks has been considered in the framework of high-density effective theory [3, 6, 14]. The particle excitation can be described by $\tilde{\psi}$ defined as $\tilde{\psi} = \frac{1}{2}(1 + \vec{\alpha} \cdot \hat{v}_F)\psi$, where ψ is the quark field and $\hat{v}_F$ is the Fermi velocity, in the effective theory [6]. Using the $\tilde{\psi}$ field, such a term as $-\frac{1}{\mu}(\tilde{\psi_L}^+ M M^+ \tilde{\psi}_L + \tilde{\psi_R}^+ M^+ M \tilde{\psi}_R)$ is introduced at the leading order in $1/\mu$ [3, 6]. In the case of $m_s \gg m_q$, this term is further simplified as

$$\mathcal{L}_q = -\frac{m_s^2}{2\mu} \tilde{\psi}^{s+} \tilde{\psi}^s. \tag{9}$$

In Ref.[6], Bedaque and Schäfer regarded $\frac{m_s^2}{2\mu}$ in Eq.(9) as the chemical potential for the strange hole excitations. According to the chemical equilibrium, the chemical potential for K^0 modes is

$$\mu_K = \frac{m_s^2}{2\mu}, \tag{10}$$

and thus the critical condition of K^0 condensation is required to be $\mu_K = m_K$. As argued in Refs.[6, 7], the CFL phase was predicted to undergo a second order transition to the CFLK^0 phase as long as $m_s^2/(2\mu)$ is larger than m_K or approximately

$$m_s \geq (\frac{2\sqrt{3}\mu}{\pi f_\pi})^{\frac{2}{3}} m_q^{\frac{1}{3}} \Delta^{\frac{2}{3}} \approx 3 m_q^{\frac{1}{3}} \Delta^{\frac{2}{3}}. \tag{11}$$

It is well known that the condensate strength is closely linked to the ratio of the kaon chemical potential and the kaon mass. With the increasing value of $m_s^2/(2\mu)$, both the density of the condensed kaons and the pressure from kaon condensation arise rapidly. For instance, if $m_s^2/(2\mu)$ is close to Δ, the condensate density has an order of $\Delta\mu^2/(2\pi^2)$ and the condensate pressure has an order of $\Delta^2\mu^2/(2\pi^2)$. Comparing with Eqs.(2) and (3), they

[1] The anomalous mass ordering is closely linked to the Lagrangian Eq.(5), where the Goldstone-mode mass term contains $det(M)M^{-1}$. If other possible terms, say the instanton interaction, were considered, this behavior would be modified. In that case, the condensations of Goldstone-modes might not occur [3, 14]. In the present work we assume that the instanton contribution to the Goldstone-mode mass is negligible at least for large quark chemical potential.

are of the same orders as the pairing contributions to the pressure and the baryon density in the CFL quark matter. In this case, the K^0 condensation should be *not merely* caused by the quasi quarks in the vicinity of the Fermi surface, which have taken part in the CFL pairing. In order to make the K^0 condensation with large condensate strength possible, it is reasonable to assume that an amount of quasi quarks residing away from the Fermi surface also take part in the process $\tilde{\psi}^{s+}\tilde{\psi}^d \rightarrow K^0$. Now that more quasi quarks than those near the given Fermi surface get involved in the K^0 condensation and the kaon chemical potential not only originates from the non-zero strange quark mass, but also comes from the contribution induced by these additional quasi quarks. In other words, the kaon chemical potential is expected to be deviated from the value given by Eq.(10) when the condensate strength is large, as will be explained in the next section.

III. Vector kaon-quark interaction and its effect on the CFLK^0 phase

According to Refs.[6, 7], the K^0 condensation is caused by the quark self-interacting term Eq.(9) exclusively. This treatment is very different from the kaon-meson condensation in hadronic environment such as in nuclear matter. As pointed out in Refs.[15], the effective interaction between baryons and kaons plays the key role in meson condensation. In the CFL environment, the quasi quarks have an energy gap Δ so that the interaction between kaon-modes and quarks is usually considered to be very small. Even if so, the effective kaon-quark interaction needs to be included in the case that the K^0 condensate strength is large enough. There are many types of kaon-quark interactions such as scalar coupling, vector coupling, pseudoscalar coupling and pseudovector coupling. For the purpose of this work [2], our main intention is to consider the leading term in the vector kaon-quark interactions

$$\mathcal{L}_{kq} = -\frac{iC}{f_\pi^2}\tilde{\psi}^+\tilde{\psi}(\overline{K}\overrightarrow{\partial_0}K - \overline{K}\overleftarrow{\partial_0}K), \tag{12}$$

where C is the coupling coefficient. Eq.(12) bears a formal analogy with the Tomozawa-Weinberg term accounting for the leading vector interaction between nucleons and kaon-mesons [15, 16]. Both Eq.(12) and the Tomozawa-Weinberg term originate from the vector coupling $\sim \overline{B}\gamma^\mu v_\mu B$, where $v_\mu \equiv \frac{i}{2}(\xi\partial_\mu\xi^+ + \xi^+\partial_\mu\xi)$ is the vector current of Goldstone bosons with $\xi = \sqrt{U} = \exp(i\Phi/\sqrt{2}f_\pi)$ and B denotes the baryon/quark field. Since we only concern the kaon mode in the Goldstone matrix, it is practical to consider the form of Eq.(12) at the

[2] We have considered the scalar interaction caused by the non-zero strange quark mass in CFL matter [17]. Since its effect on K^0 condensation is found to be small, we will ignore it in this work. As for the pseudoscalar and pseudovector interacting terms, their effects can be neglected at the mean-field level as well as in the limit of isospin symmetry .

mean field level. In addition, Eq.(12) is valid even in the chiral limit and it is responsible for the dynamical breaking of chiral symmetry. Noticing that Eq.(9) is responsible for the explicit chiral breaking, thus incorporation of Eqs.(9) and (12) is necessary for reflecting the symmetry breaking pattern in the CFL context.

When K^0 condensation occurs, the expectation values of the neutral kaon and anti-kaon fields can be expressed as [6, 15]

$$\langle K^0\rangle = \frac{f_\pi}{\sqrt{2}}\theta\exp(-i\mu_K t), \quad \langle\overline{K}^0\rangle = \frac{f_\pi}{\sqrt{2}}\theta\exp(i\mu_K t), \tag{13}$$

respectively, where θ is a dimensionless parameter determining the condensate strength. It is easy to find that the expectation value of $i(\overline{K}\overrightarrow{\partial_0}K - \overline{K}\overleftarrow{\partial_0}K)$ can be replaced by the condensate density denoted as ρ_{con} and then Eq.(12) becomes

$$-\frac{C\rho_{con}}{f_\pi^2}\tilde{\psi}^+\tilde{\psi}, \tag{14}$$

at the mean field level. Comparing Eq.(14) with Eq.(9), it means that $C\rho_{con}/(f_\pi^2)$ can be regarded as an additional effective chemical potential for the strange hole excitations. As expected, there is a shift in the kaon chemical potential due to the presence of K^0 condensation.

To treat Eq.(14) self-consistently, we need to replace μ_K by $\mu_K{}'$ as

$$\mu_K \to \mu_K{}' = \frac{m_s^2}{2\mu} + \frac{C\rho_{con}}{f_\pi^2}, \tag{15}$$

into the calculations for K^0 condensation. Then, the condensate density and the condensate pressure take the forms :

$$\rho_{con} = f_\pi^2\mu_K{}'[1 - \frac{m_K^4}{(\mu_K{}')^4}], \tag{16}$$

and

$$\mathcal{P}_{con} = \frac{1}{2}f_\pi^2(\mu_K{}')^2[1 - 2\frac{m_K^2}{(\mu_K{}')^2} + \frac{m_K^4}{(\mu_K{}')^4}], \tag{17}$$

respectively, instead of those given previously [7, 10]. Eqs.(16) and (17) are valid only for the case that K^0 have condensed.

This is not the whole story yet. The quasi quarks residing away from the Fermi surface could not become active unless they gain pressure from that induced by the CFL pairing. Therefore, the K^0 condensation should affect the pairing phenomena in the CFL matter in turn. As pointed out by Alford *et. al.* [12], the CFL pairing gap is unchanged approximately as long as $m_s^2/(2\mu)$ is still smaller than Δ. In this case, the K^0 condensation can affect the

pairing pressure via changing the magnitude of μ in Eq.(2) equivalently. In view of the fact that the shift in the kaon chemical potential Eq.(15) is a sort of Fermi energies essentially, we simply assume that the " effective " quark chemical potential is suppressed by the same shift, namely $\frac{C\rho_{con}}{f_\pi^2}$. Here we stress that the actual quark chemical potential is still determined by the baryon density of the unpaired quark matter, so that the definition of Eq.(4) is not influenced by the K^0 condensation. In other words, the decrease of the " effective " quark chemical potential makes sense only for the pairing phenomena in the CFL matter. Therefore, the pairing pressure Eq.(2) and then the baryon density Eq.(3) are replaced by

$$\mathcal{P}_{pair} = \frac{3\Delta^2(\mu - \frac{C\rho_{con}}{f_\pi^2})^2}{\pi^2}, \tag{18}$$

and

$$\rho_{CFLK^0} \;\; = \;\; \frac{p_F^3 + 2\Delta^2(\mu - \frac{C\rho_{con}}{f_\pi^2})}{\pi^2}, \tag{19}$$

respectively.

Eq.(19) is the baryon density of the CFL matter in which kaons condense. It implicates that the baryon density in the CFLK^0 phase is reduced more or less, which is an indirect result of the K^0 condensation effect on the CFL free energy. This observation is different compared to the previous scenario, where the baryon density of CFL matter is independent of whether K^0 condensation occurs or not. More importantly, the pressure in the CFLK^0 phase is different from the previous form. Due to Eq.(18), the formation of K^0 condensation makes the pairing pressure to be smaller than that in the CFL phase. On the other hand, the K^0 condensation provides a contribution to the pressure of the CFLK^0 phase. With the increasing condensate strength, the shift in the kaon chemical potential is enlarged so that the kaon-condensate pressure is reinforced to be larger. The total pressure of the CFLK^0 phase reads

$$\mathcal{P}_{CFLK^0} = \mathcal{P}_{unpair} + \mathcal{P}_{pair} + \mathcal{P}_{con}, \tag{20}$$

where the second term of RHS is defined in Eq.(18).

As long as the pressure in the CFLK^0 phase is smaller than that in the CFL phase, the CFLK^0 phase is no longer energetically favorable. A stable CFLK^0 phase does not exist until the pressures of the CFL and CFLK^0 phases reach a new equilibrium. In this sense, the critical condition of the formation of the stable CFLK^0 phase becomes

$$\mathcal{P}_{CFLK^0}(m_s, \mu) = \mathcal{P}_{CFL}(m_s, \mu). \tag{21}$$

In fact, Eq.(21) is just the Gibbs condition for the first order phase transition. Therefore, the CFL phase should undergo a first-order transition, rather than the second-order one, to the stable CFLK^0 phase.

IV. Numerical results and discussions

Finally we present some numerical examples to demonstrate how the modified mechanism works. For sufficiently large μ, the description of the Goldstone modes in the CFL matter is valid. In such a perturbative regime, nevertheless, the kaon condensation effects become negligible since $m_s^2/(2\mu)$ is very small. We may extrapolate the CFL matter from the perturbative regime to the regime with $\mu \sim 1$GeV. For the range of $\mu = 0.5 - 1.5$GeV, we adopt the current estimate $\Delta \sim 100$MeV [18]. Through this work, m_s is treated as a parameter whereas the quark mass ratio is set as $m_s/m_q \sim 20$ [19]. As for the constant C in Eq.(12) we use some typical values, which are small enough but are of order one [3] , in the numerical calculations. At a given quark chemical potential $\mu = 1$GeV, Fig.1 demonstrates the dependence of the pressures for the two phases of the CFL matter on the strange quark mass, while Fig.2 shows the dependence of the baryon densities on m_s. At $m_s \approx 120$MeV, Eq.(11) is satisfied and K^0 condensation comes to occur. With increasing m_s, nevertheless, the pressure of the CFLK^0 phase becomes smaller than that of the CFL phase. In this case, the K^0 condensed phase is energetically unstable in the CFL matter. Also, the baryon densities in the CFLK^0 and CFL phases separate from each other for $m_s > 120$MeV, as shown in Fig.2. When m_s is close to 380MeV the CFLK^0 and CFL phases reach a pressure equilibrium and the CFLK^0 phase gradually becomes energetically favorable as long as m_s is larger than this value. So there exists a first order CFL-CFLK^0 phase transition at $m_s \approx 380$MeV. Also a discontinuous jump of the baryon density takes place at this point, as shown by the vertical dotted line in Fig.2.

With different values of C, a schematic phase diagram of CFL matter is given in Fig.3. The critical line of the CFL-CFLK^0 transition is obtained from Eq.(21), while the boundary of CFL matter is described by a line $m_s^2/(2\mu) = 2\Delta$ at which the CFL pairing is predicted to break down completely [1, 6]. Also the gapless CFL phase is shown in Fig.3, which is predicted to become dominant for the regime of $m_s^2/(2\mu) > \Delta$ [12]. When $C = 0$, namely the effective

[3] If the vector kaon-quark coupling is introduced in the framework of high-density effective theory, C should be of order one by power counting. If it is introduced in the framework of the CFL effective theory with two scales f_π and Δ, the coupling of quark and vector current v_μ may have different coefficients. As argued in Ref.[20], such a coefficient scales as f_π/Δ by power counting. In this case, C in Eq.(12) should be still of order one for our concerned regime. But the above estimates by the effective theory could not be taken too seriously since our treatment of Eq.(12) only holds at the mean field level.

kaon-quark interaction is ignored, the critical line of the CFL-CFLK^0 transition coincides with that given in the literature [6, 7]. In this case, our mechanism for K^0 condensation comes back to the previous one, as shown by the lowest dashed line in Fig.3. With increasing C, the stable CFLK^0 phase is limited to a smaller window in the (m_s, μ) plane. As long as C is not very small, there is not too much room for the CFLK^0 phase in the (m_s, μ) plane. Depending on the coupling coefficient of kaon-quark interaction, the K^0 condensed phase might be energetically disfavored compared to the normal CFL phase. The present result is different from that drawn by including the instanton effect, which raises the masses of Goldstone-modes from the values given by Eq.(8) and thus makes K^0 condensation difficult to occur [8, 14]. As a starting point, we assume that the masses of Goldstone-modes are determined by Eq.(8) at least for $\mu \sim 1$GeV. Thus the formation of K^0 condensation is still triggered by the non-zero quark masses. Once the condensate density becomes finite, the kaon-quark interaction works. Therefore the formation of the stable CFLK^0 phase, i.e. the CFL-CFLK^0 transition, is of first order generally. If the instanton interaction is included, this conclusion still holds although the CFLK^0 phase is more difficult to appear. For the regime with a relatively small μ, a systematic analysis on this issue needs to consider dependency of the common Δ on μ and the color-sextet-channel pairing too, which might change our numerical results quantitatively.

In summary, the mechanism for K^0 condensation and the stability of the CFLK^0 phase are examined in the phase region between the normal CFL phase and the gapless CFL phase. Taking the vector kaon-quark interaction into account, we find that the CFL pairing phenomena of dense quark matter change gradually as m_s is chosen to be larger. This conclusion is essentially consistent with the trend that the CFL pairing, say that between the green-strange and blue-down quarks, becomes unstable with increasing strange quark mass [12]. Another interesting point is that the CFLK^0 phase is predicted to have the lower baryon density than the normal CFL phase. It may be important for understanding the phenomena in the QCD phase diagram. For instance, if the baryon density and the strangeness density in the stable CFLK^0 phase become comparable with those in the hypernuclear phase under somewhat conditions, one could not rule out the possibility of the so-called hadron-quark continuity [9, 11]. Further investigations on how the CFLK^0 phase, if it exists as a stable state, undergoes a transition(s) to the gapless CFL and/or non-CFL phases need to use model-dependent descriptions of the various CFL phases and consider effects of the electron and color chemical potentials. Works along these directions are worth being pursued.

Acknowledgements

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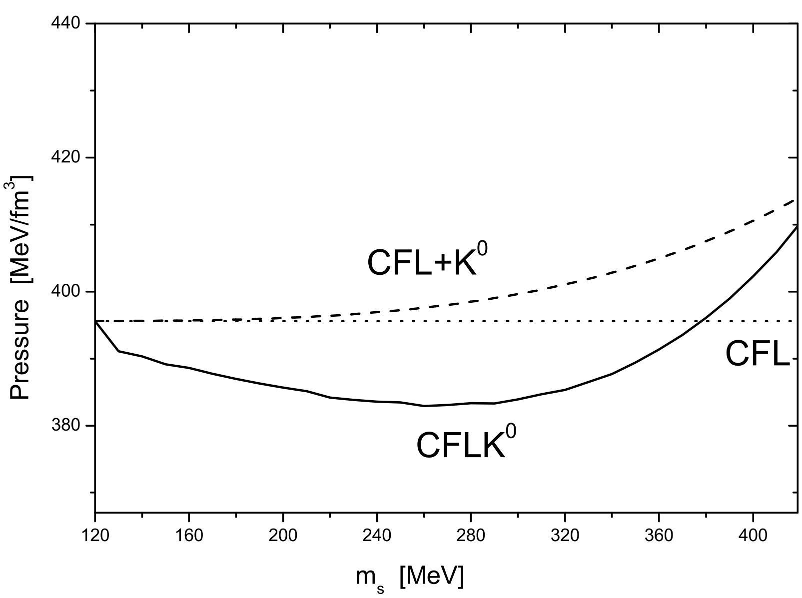
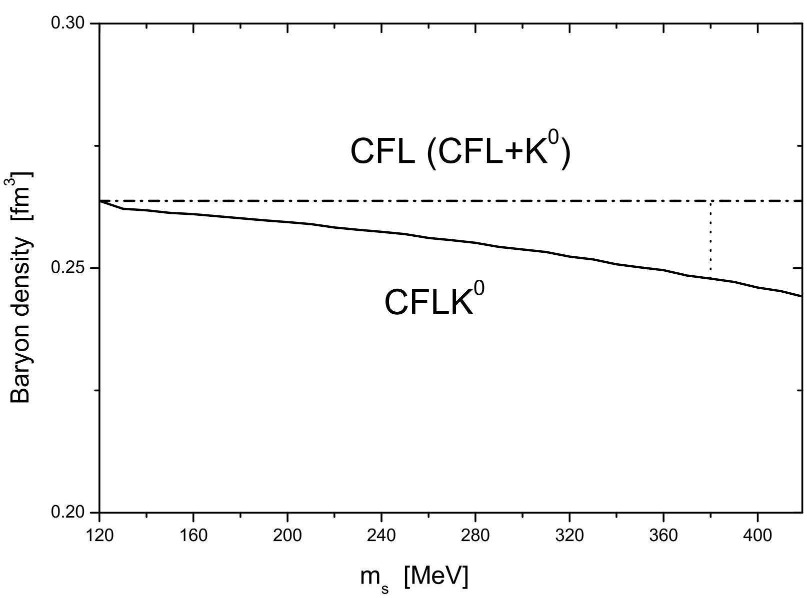
Figure 1: Pressures for the CFLK^0 (solid) and CFL (dotted) phases as functions of strange quark mass at $\mu = 1$GeV with the choice of $C = 0.5$, where the contributions from the unpaired quark matter to pressures are subtracted. The dashed line corresponds to the previous result, where the properties of CFL matter are not influenced by K^0 condensation.

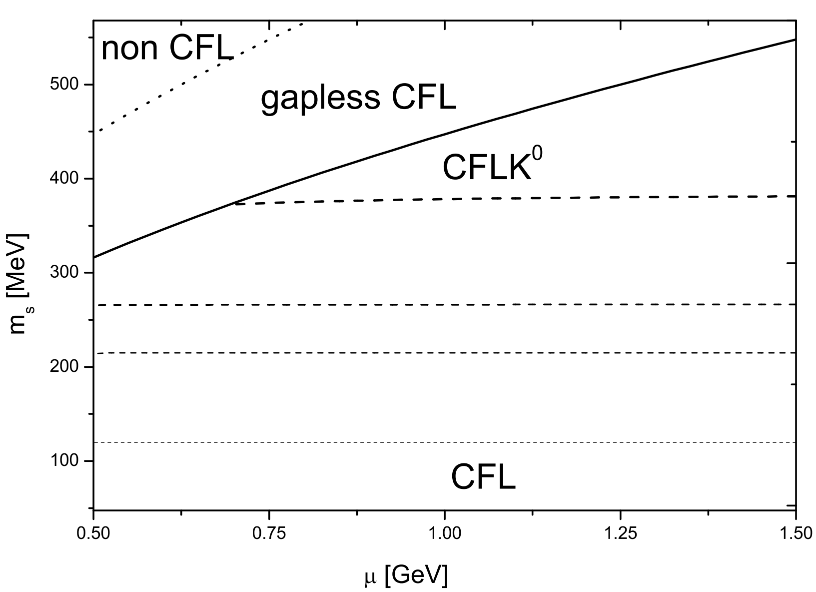
Figure 2: Similar as Fig.1, but for the baryon densities. The contributions from the unpaired quark matter are subtracted also.

Figure 3: Schematic phase diagram of CFL matter. The dashed lines from up to bottom indicate the CFL-CFLK^0 phase transitions with $C = 0.5, 0.1, 0.05, 0$. The solid line denotes the formation of the gapless CFL phase, whereas the dotted line does the boundary of CFL quark matter.



440
420
400
380
Pressure [MeV/fm^3]
120
160
200
240
280
320
360
400
m_s [MeV]
CFL+K^0
CFL
CFLK^0


CFL (CFL+K^0)
CFLK^0
Baryon density [fm^3]
0.30
0.25
0.20
120
160
200
240
280
320
360
400
m_s [MeV]



non CFL
gapless CFL
CFLK^0
CFL
m_s [MeV]
μ [GeV]
100
200
300
400
500
0.50
0.75
1.00
1.25
1.50